Decomposition of a Train of KdV Solitons by Hilbert-Huang Transform

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Introduction

Large Amplitude internal solitons are ubiquitous in marginal tropical oceans such as: Sulu Sea, Caribbean Sea, Andaman Sea. In locations such as the northern South China Sea and the Mediterranean Sea have been well studied. Large negative amplitude solitons have been well represented by KdV solitons. In this paper we are interested in simulate an oceanic soliton train using a KdV soliton train. We designed various cases of soliton trains by means of changing their amplitude and time spacing. The KdV soliton train is a nonlinear system and can be analyzed using a novel technique known as Hilbert-Huang Transform. Each case was analyzed separately. This method allows us to extract information about the forcing of the system and understand its nonlinear nature.

Methodology

We designed a three KdV soliton train represented with the following equation:

$u = -\frac{1}{2} c_1 \operatorname{sech}^2 [(\sqrt{c_1})(x-3)/2] -\frac{1}{2} c_2 \operatorname{sech}^2 [(\sqrt{c_2})(x-10)/2] -\frac{1}{2} c_3 \operatorname{sech}^2 [(\sqrt{c_3})(x-17)/2]$

The above expression was evaluated in a linearly spaced vector **x** delimited by zero and 20 and partitioned in 256 points. The time vector was obtained by multiplying the vector **x** times **dt**, the assigned time interval. By adjusting **dt** the time interval between KdV solitons is changed. Below is a table with the input parameters for five different cases:

Case	dt	Soliton Period (min.)	c_1,c_2,c_3
1	7.0	49.1	-16, -16, -16
2	3.5	24.6	-16, -16, -16
3	3.5	24.6	-16, -14, -12
4	1.3	12.3	-16, -16, -16
5	2.6	18.3	-16, -16, -16

Figure 1 shows the first three cases. The first case shows three equal amplitude solitons separated by 49.1 minutes. The soliton train spans 0.1 day (2.4 hours). The second case has half the time span 0.05 day (1.2 hours) but the solitons are separated by 24.6 minutes. For the third case we kept the time span but changed to unequal amplitudes. Soliton in nature tend to follow a similar order. The large leader soliton followed by the smaller ones.

Each soliton train was subjected to the Empirical Mode Decomposition (EMD), a novel method to analyze nonstationary and nonlinear signals (Huang et al. 1998). This adaptive method has been applied to the analysis nonlinear water waves (Huang et al. 1999). From EMD we obtain intrinsic mode functions (IMF) components that have a physical interpretation. Well known nonlinear systems such as the Duffing equation has been subject to Hilbert Huang Transform (HHT) analysis and each component have a physical meaning. Using Normalized Hilbert Transform (NHT) we found the instantaneous frequencies of each IMF component obtained by EMD. The concept of instantaneous frequency has been well explained by Huang et al. (2009).

This analysis was performed in Matlab, using the m-files provided by: 中央大學數據分析中 Research Center for Adaptive Data Analysis. Chungli,Taiwan http://rcada.ncu.edu.tw/intro.html

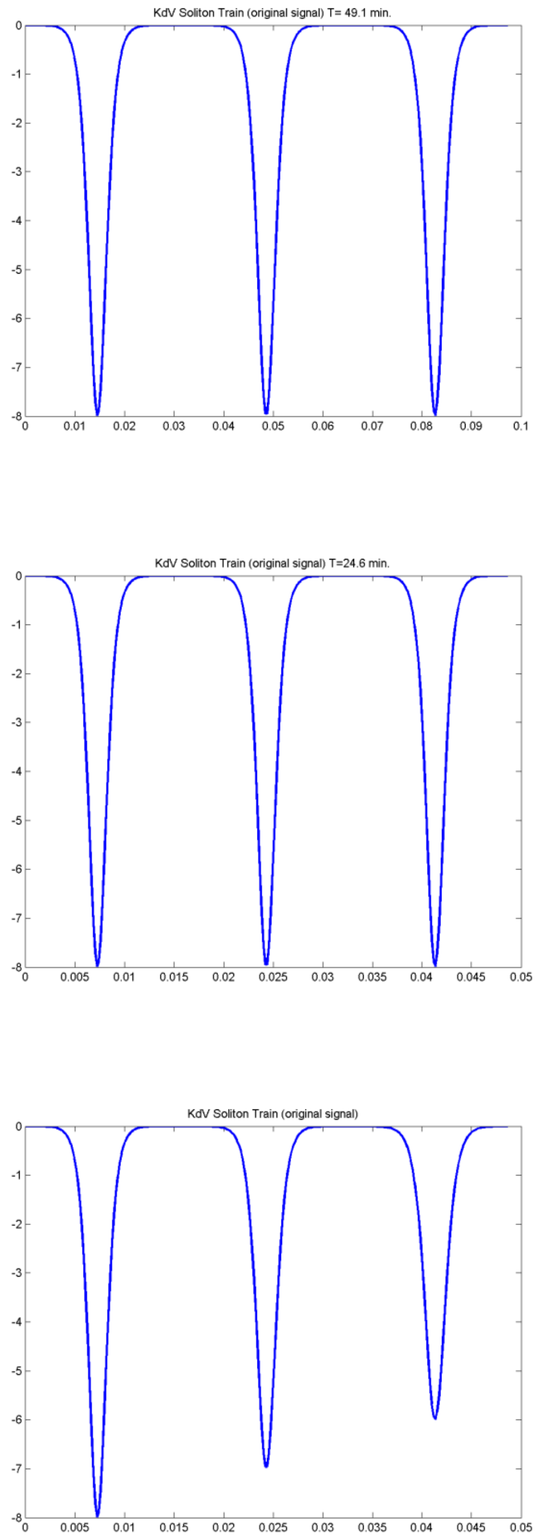

Figure 1. Three trains of KdV negative polarity solitons with two different time scales: 49.1 min. (top), 24.6 min. (middle and bottom). In the last train we kept the time scale but change the amplitudes (8, 7 and 6 m).

Results

When subjected to the Empirical Mode Decomposition (EMD), the KdV soliton train yields only two prominent IMF components (C1, C2) as shown in Figure 2. C1 emulates the nonlinear shape of the original soliton train, but is offset up by four units. C1 represents most of the nonlinear part of the signal. For the case of equal amplitude solitons, the shape of C2 is symmetric and C9 is flat offset at minus four units. However, for the case of unequal amplitudes, C2 lost symmetry and C9 showed a positive slope trend. The trend compensates for the declining C1. Components C3 to C8 were not shown because they have insignificant magnitudes of the order of 1E-16. The sum of C1, C2 and C9 results in the original soliton train.

C1 and C2 were subjected to the Normalized Hilbert transform (NHT) and their instantaneous frequencies are shown in Figure 3. For all cases, the instantaneous frequency of C1 was clearly nonlinear. For the 49.1 min. soliton train the instantaneous frequency ranged between 0.36 and 10.7 cycles per day (CPD). For the 24.6 min soliton train, ranged between 0.78 and 21.44 CPD. Doubling the frequencies of the first case. C2's instantaneous frequencies are practically linear, except at the beginning and end, affected by the poor spline fitting at both ends of the signal. When the soliton amplitudes were unequal, small oscillations appeared. For the 49.1 min. soliton train the instantaneous frequency values of C2 ranged between 0.74 and 1.19 CPD with a mean of 0.97 CPD. For the 24.6 min soliton train, ranged between 1.49 and 2.37 CPD with a mean of 1.93 CPD. The above results were summarized below in Table 1. A closer view to the data in Table 1 reveals that each time we reduce by half the period of the soliton train (but keep the amplitudes equal) the instantaneous frequencies of C1 and C2 are doubled.

These results revealed that C1 represents the nonlinear component of the train and C2 the quasi linear component. C1 shows the intrinsic nonlinear nature of the signal and C2 represents the quasi linear nature of the forcing. Both components have a physical interpretation.

Table 1. Normalized Hilbert Transform (NHT) results for five cases.

CASE	KdV Soliton Train Period (Minutes)	KdV Soliton Train Amplitudes (m)	C1's Instantaneous Frequency (CPD)		C2's Instantaneous Frequency (CPD)		
			MIN	MAX	MIN	MAX	MEAN
1	49.1	-8, -8, -8	0.36	10.7	0.74	1.19	0.97
2	24.6	-8, -8, -8	0.78	21.44	1.49	2.37	1.93
3	24.6	-8, -7, -6	0.69	21.25	2.08	2.2	2.14
4	12.3	-8, -8, -8	1.56	42.88	2.98	4.74	3.86
5	18.3	-8, -8, -8	1.00	28.6	2.03	3.18	2.61

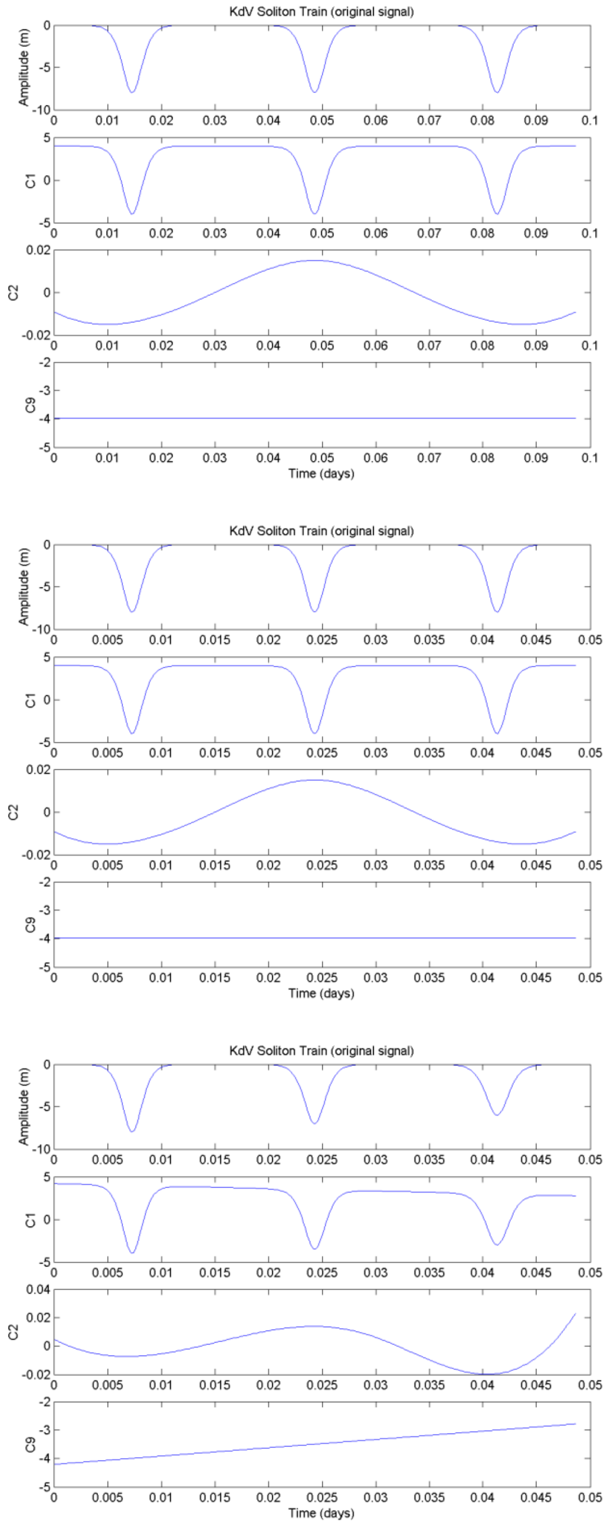

Figure 2. IMF's obtained from the EMD analysis of the respective KdV soliton trains. (Top and Middle) We obtained two IMF's (C1 and C2) and a flat offset (C9). (Bottom) For the last case we obtained a trend (C9). IMF's C3 to C8 were omitted due to their insignificant magnitude (Order 1E-16).

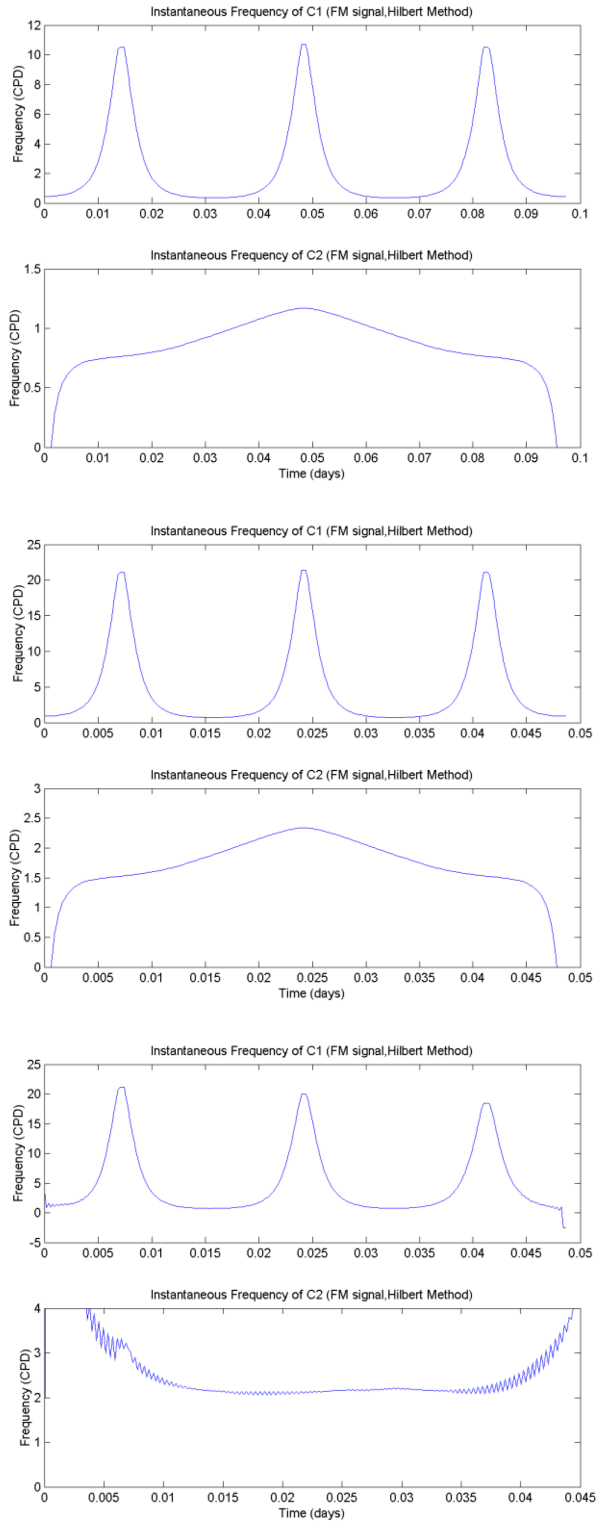

Figure 3. Instantaneous Frequency of the IMF’s: C1 and C2; for each KdV soliton train case.

Discussion

The above results show that EMD can indeed analyze a nonlinear soliton train. The train was decomposed into two prominent IMF components: C1 and C2. The first IMF is a uniform amplitude high frequency component. C1 reveals the true nonlinear nature of the train. C1's instantaneous frequency shows intrawave frequency modulation. The maximum instantaneous frequency occurred at the time of maximum negative soliton amplitude. The second IMF is a small amplitude low frequency component and shows a symmetric sinusoidal shape when the soliton amplitudes are kept equal. C2's instantaneous frequency is not strictly linear but shows a quasi linear trend. We believe that both components can be interpreted physically. KdV solitons trains are an intrinsic nonlinear phenomena and C1 revealed its nature. C2 revealed the quasi linear nature of the system forcing. Similar interpreted IMF's have been obtained in the past from HHT analysis of nonlinear systems such as the Duffing equation (Huang et. al., 1998).

Negative amplitude internal solitons have been studied in tropical seas such as: the Sulu Sea, South China Sea and Caribbean Sea. Physical oceanographers have demonstrated that the observed internal solitons can be truly represented by KdV solitons, especially in the continental slope waters. Periods between 10 and 100 minutes separating ocean internal solitons are commonly reported. In our study, the KdV soliton train was designed with similar time scales: 24.6 and 49.1 minutes. Table 1 show that C1's instantaneous frequency reaches a maximum value of 21 CPD for the case of a 24.6 minutes soliton train. By the same reasoning, C1's instantaneous frequency must have a physical interpretation. The Parguera Shelf Response resonates at 28 CPD when excited by internal solitons. It is possible that a soliton train with amplitude of 15 m with maximum C1's instantaneous frequency of 28.0 CPD and a time scale of 24.6 minutes could be responsible for the coastal seiche oscillations at La Parguera (See Appendix). C2's mean instantaneous frequencies near 2 CPD suggest that the system forcing is semidiurnal in nature. Large amplitude solitons are generated by strong semidiurnal currents during perigean spring tides. The semidiurnal constituents M2 and S2 have a frequency of 1.93 CPD and 2 CPD, respectively. A 24.6 minutes soliton train is linked to semidiurnal forcing. Similarly, C2's mean instantaneous frequency revealed that a 49.1 minutes soliton train is linked to diurnal forcing.

Conclusion

EMD analysis successfully decomposed an internal KdV soliton train into a nonlinear component and a quasi linear component. NHT analysis demonstrated that the second component represents the forcing of the soliton train. The instantaneous frequency of C2 reveals if the soliton train is forced by semidiurnal or diurnal tidal currents. C1's instantaneous frequency showed intrawave frequency modulation. The maximum instantaneous frequency occurred at the time of maximum negative soliton amplitude.

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Appendix: Ideal Cases of KdV Soliton Trains for a Near Resonant Parguera Shelf Response.

The maximum shelf response at La Parguera occurs at the non-dimensional frequency of 0.105 equal to 28.8 CPD as shown in Figure 4 (Chapman & Giese, 1990; Alfonso-Sosa, 2011). A KdV soliton train impinging near the shelf break can excite a resonant shelf response if C1's Maximum Instantaneous Frequency reaches the 28.8 CPD.

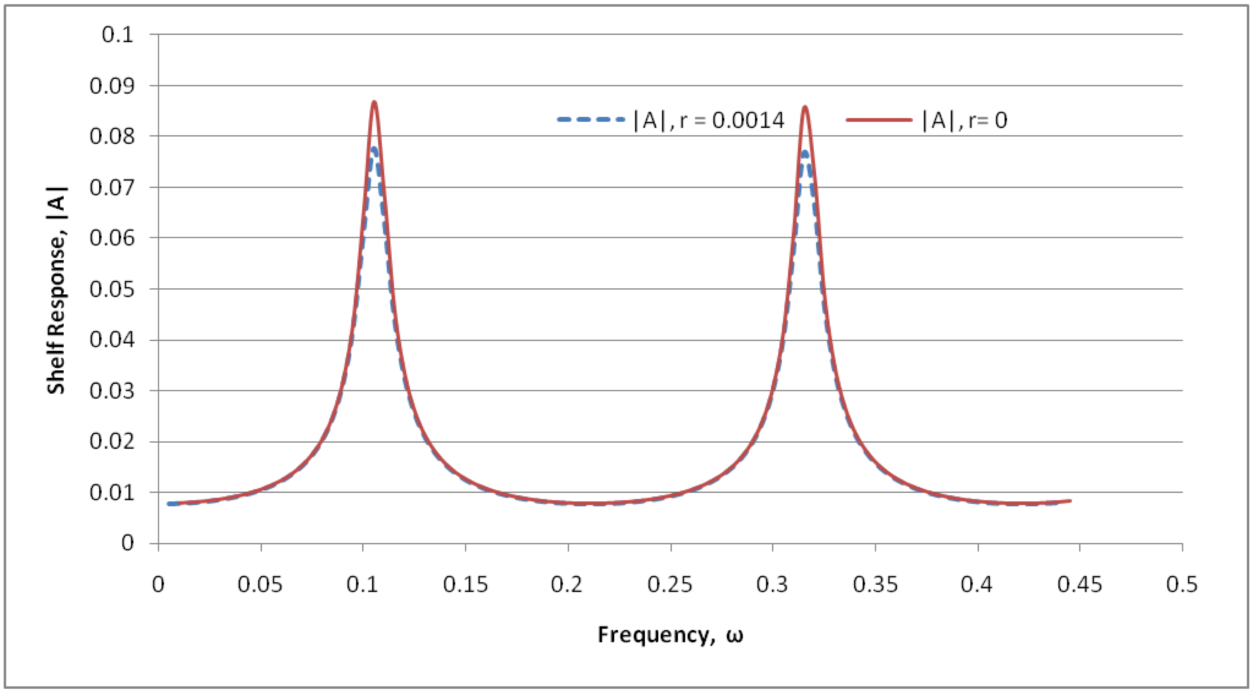

Figure 4. Analytical Model of Chapman & Giese (1990) shows the response of the insular platform, | A |, on the outskirts of La Parguera, PR based on the non-dimensional frequency, ω, to the excitation of internal waves from deep water. Two prominent peaks appear 0.105 in (28.8 CPD) and 0.316 (86.4 CPD). The blue dashed line represents the response of the platform when we include bottom friction.

Large amplitude coastal seiches at La Parguera should occur when C1's Maximum Instantaneous Frequency value is close to 28 CPD. But this value depends on both: the KdV soliton train period and the KdV soliton amplitude. Table 2 shows the Maximum C1's Instantaneous Frequencies for various cases of KdV soliton trains. Case 1, 2 and 5 show the adequate match pairs of soliton period and amplitude: (-49.1 min, -120 m), (24.6 min, -15 m) and (18.3 min, -8 m), respectively. A shorter period soliton train with smaller amplitude can reach the 28 CPD, in contrast we need a larger amplitude for a longer period soliton train.

Table 2. Maximum C1's Instantaneous Frequencies for various cases of KdV soliton trains

CASE	KdV Soliton Train Period (Minutes)	KdV Soliton Train Amplitudes (m)	C1's Instantaneous Frequency (CPD)
			MAX
1	49.1	-120, -110, -100	27.9, 29.5, 26.8
2	24.6	-18, -17, -15	30.1, 30.5, 28.0
3	24.6	-14, -13, -12	27.4, 26.8, 25.4
4	12.3	-5, -4, -3	33.6, 30.1, 27.0
5	18.3	-8, -7, -6	28.2, 26.8, 24.9

Only with the correct match of period-amplitude we can obtain a Maximum C1's Instantaneous Frequency of 28 CPD. Table 3 summarizes the matched pair values for various cases.

Table 3. KdV soliton train amplitude versus the KdV soliton train time period for La Parguera Shelf Response Frequency at 28 CPD.

CASE	KdV Soliton Train Period (Minutes)	KdV Soliton Train Period (Seconds)	KdV Soliton Train Amplitudes (m)	C1's MAX Instantaneous Frequency (CPD)
4	12.3	738	-3.5	28.34
5	18.3	1098	-8	28.2
2	24.6	1476	-15	28
9	27.9	1674	-20	27.7
6	35.2	2112	-37	28.2
10	38.4	2304	-50	28.34
7	42	2520	-59	28.6
8	45.5	2730	-86	28.1
1	49.1	2946	-120	27.9

From Table 3 data we constructed the graph shown on Figure 5. This graph represents the ideal soliton trains to generate large amplitude coastal seiches at La Parguera. An exponential trendline was adjusted to the data with an R=0.99. The following equation allows us to find the ideal KdV soliton amplitude for an ideal KdV soliton train period to excite coastal seiches at La Parguera:

$$y = -1.692\, e^{0.08654x},$$

where y is the KdV soliton amplitude in meters and x is the KdV soliton train period in minutes.

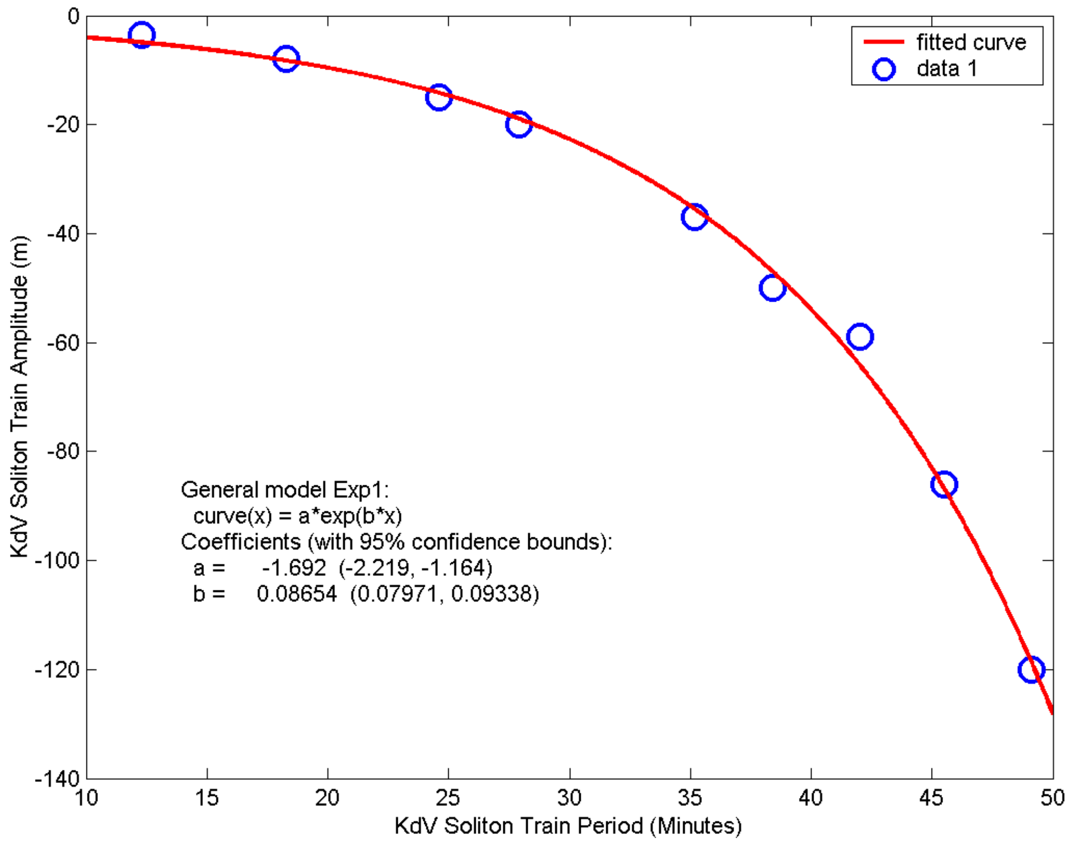

Figure 5. Graph based on data from Table 3.