Comparison Between the Instantaneous Frequencies of the Offshore KdV Soliton Train and of the Coastal Shelf Response

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Introduction

Large Amplitude internal solitons are ubiquitous in marginal tropical oceans such as: Sulu Sea, Caribbean Sea, Andaman Sea. In locations such as the northern South China Sea and the Mediterranean Sea have been well studied. Large negative amplitude solitons have been well represented by KdV solitons. In this paper we are interested in simulate an oceanic soliton train using a KdV soliton train. We designed two cases of soliton trains by just changing their amplitude. The KdV soliton train is a nonlinear system and can be analyzed using a novel technique known as Hilbert-Huang Transform. Each case was analyzed separately. This method allows us to extract information about the forcing of the system and understand its nonlinear nature. Each soliton train was subjected to the Empirical Mode Decomposition (EMD), a novel method to analyze nonstationary and nonlinear signals (Huang et al. 1998). This adaptive method has been applied for the analysis of nonlinear water waves (Huang et al. 1999). From EMD we obtain intrinsic mode functions (IMF) components that have a physical interpretation. Well known nonlinear systems such as the Duffing equation has been subject to Hilbert Huang Transform (HHT) analysis and each component have a physical meaning. Using Normalized Hilbert Transform (NHT) we found the instantaneous frequencies of each IMF component obtained by EMD. The concept of instantaneous frequency has been well explained by Huang et al. (2009). The Normalized Hilbert transform (NHT) method consists of applying the Hilbert transform to the empirically determined FM signals. From a previous work (Alfonso-Sosa, 2013), we know that the KdV soliton train's instantaneous frequency can change due to both changes in the soliton's amplitude and soliton train's period.

In addition, we calculate the shelf response for an identical pair of incident internal KdV solitons. The incident interfacial KdV soliton train generates a surface pulse at the shelf break which travels across the shelf toward the coast generating a surface shelf response, a.k.a. coastal seiche (Chapman and Giese, 1990). The coastal seiche is a nonlinear surface wave which exhibits wave distortion and its instantaneous frequency can be analyzed by means of NHT. Our study focus in comparing changes in the instantaneous frequency of both nonlinear signals: offshore internal soliton train and coastal seiche. We try to understand how the system is coupled. How changes in soliton train's instantaneous frequency are reflected as changes in shelf response's instantaneous frequency. For this study, we fixed the soliton train's period (100 minutes) but doubled the soliton's amplitude to induce changes in both signals.

Methodology

To generate the KdV soliton train and the subsequent coastal shelf response we applied a modified version of the analytical model for La Parguera Shelf developed by Chapman and Giese (1990). It is a linear two-layer coastal model in which internal waves from the deep ocean impinge upon step-shelf bottom topography. The model modification consists in substituting the two incident Gaussian pulses by two KdV solitons. In addition, the time period between the solitons was increased from 50 minutes to 100 minutes; this way eliminates any possible interaction between them. The same input parameters used by Chapman and Giese were preserved in this model. The model is contained in a public available m-file (MATLAB) function developed by Ocean Physics Education Inc. and can be downloaded from Mathworks at the following address:

We use the m-file function called shelfresponse2KdV:

http://www.mathworks.com/matlabcentral/fileexchange/44603-shelf-response-for-two-kdv-solitons/content/shelfresponse2KdV.m

Soliton KdV equations are based on a two-layer model from Apel et al. (2006):

$$\eta(x,t) = \eta_0 \ \mathrm{sech}^2 \frac{x - Vt}{\Delta},$$

where η_0 , is the soliton amplitude. V and Δ are the nonlinear speed, and soliton width parameter, respectively. The nonlinear speed V is faster than the linear speed **c**. Defined below as:

$$V = c + \frac{\alpha \eta_0}{3}, \quad \Delta^2 = \frac{12\beta}{\alpha \eta_0}.$$

where α and β are the environmental parameters defined below as:

$$c = \left[\frac{g(\rho_2 - \rho_1)h_1 h_2}{\rho_2 h_1 + \rho_1 h_2}\right]^{1/2} \simeq \left[\frac{g\delta\rho}{\rho_m}\frac{h_1 h_2}{h_1 + h_2}\right]^{1/2},$$

$$\alpha = \frac{3c}{2h_1 h_2}\frac{\rho_2 h_1^2 - \rho_1 h_2^2}{\rho_2 h_1 + \rho_1 h_2} \simeq \frac{3}{2}c\frac{h_1 - h_2}{h_1 h_2},$$

$$\beta = \frac{c h_1 h_2}{6}\frac{\rho_1 h_1 + \rho_2 h_2}{\rho_2 h_1 + \rho_1 h_2} \simeq \frac{c h_1 h_2}{6}.$$

where ρ_1, ρ_2, h_1, h_2, represent the upper layer density, lower layer density, upper layer thickness, lower layer thickness, respectively.

The following table summarizes the input parameters for the shelfresponse2KdV function:

Argument	Description	Input Value	Unit
to	Initial Time in minutes	-100	minutes
tf	Final Time in minutes	600	minutes
dt	Time Interval in minutes	1	minutes
rho1	Upper layer density	1024	kg/m^3
rho2	Lower layer density	1027	kg/m^3
eta_i	Amplitude of KdV Soliton	22.5	meters
tshift*	Time shift for the second pulse. Shift to the right side of time axis (later time).	100.34	minutes
S	Shelf Depth	18	meters
L	Length of Shelf	10000	meters
H1	Deep-Ocean Upper Layer Depth	128	meters
H	Deep-Ocean Total Depth	4000	meters

*The argument tshift determines the soliton train period.

The output parameters for the shelfresponse2KdV function are: zeta_pulse, eta_s and u_s representing the internal KdV soliton train, the coastal shelf response and the current at the shelf break, respectively. The first two parameters were subject to EMD analysis to obtain their respective intrinsic mode functions (IMF). The first IMF represents the nonlinear part of the signal. Using Normalized Hilbert Transform (NHT) we found the instantaneous frequencies for the first IMF component. We applied to methods: Hilbert and Quad.

This analysis was performed in Matlab, using the m-files provided by: 中央大學數據分析中 Research Center for Adaptive Data Analysis. Chungli,Taiwan http://rcada.ncu.edu.tw/intro.html

Results

Figure 1 (top panel) shows the KdV soliton train. It consists of two equal amplitude KdV solitons separated by 100.34 minutes. The soliton train spans 0.1545 days (222.45 minutes). The soliton amplitude is -22.5 meters. The negative amplitude peaks are situated at 0.0348 days (50.17 minutes) and 0.1045 days (150.51 minutes). The middle panel shows the first IMF (C1) obtained from the empirical mode decomposition (EMD) of the KdV soliton train. C1 emulates the nonlinear waveform of the original soliton train, but is offset up by eleven units. C1 reveals the true nonlinear nature of the train. The bottom panel shows C1's instantaneous frequency and its intrawave frequency modulation. The frequency range goes from 5 CPD to 30 CPD. The maximum instantaneous frequency reaching 30 CPD occurred at the exact time of maximum negative soliton amplitude. The first IMF represents the nonlinear component of the signal.

Figure 2 (top panel) shows the coastal shelf response after been excited by the KdV soliton train. The coastal seiche spans 0.4234 days (610 minutes). The first major peak is situated at 0.0079 days and the largest peak at 0.0779 days. The time period between both is 0.0700 days (100.8 minutes). This time interval is equivalent to the KdV soliton train period. The third peak jump in amplitude (25 cm) is due to the arrival of the second KdV soliton. After the second soliton excitation the shelf response amplitude starts decaying. The middle panel shows the first IMF component (C1) of the shelf response. The bottom panel shows C1's instantaneous frequencies affected by the poor spline fitting at both ends of the signal. Despite that, the rest of the signal shows a clear intrawave frequency modulation. A large peak of 29.9 CPD in the instantaneous frequency at 0.0783 days corresponds exactly with the arrival of the second soliton (0.0779 days). This sudden increase in the shelf response's instantaneous frequency revealed the excitation by the second KdV soliton. After this event the peak values kept under the 29.5 CPD.

To limit the spline edge effects on the instantaneous frequency we applied the Quad Method (quadrature method) on both signals. Figure 3 upper panel shows C1's instantaneous frequency for the KdV soliton train. It is practical identical to the one determined through the Hilbert Method (Figure 2, bottom panel). Figure 3 bottom panel shows C1's instantaneous frequency for the shelf response obtained by the Quad Method. It is less noisy than its Hilbert counterpart. Some of the peaks surpass the 30 CPD level. The peak situated at 0.1657 days occurred just after the soliton train signal ends.

Figure 4 shows the comparison between C1's instantaneous frequencies once doubled the soliton amplitude from 22.5 m to 45 m. Hilbert Method (Top) and Quad Method (Bottom). At the foot of Figure 4 there is a table which compares the peak instantaneous frequencies values at 0.036 days. Doubling the amplitude increased the instantaneous frequency in average by 15.34 CPD. Repeating the same exercise (Figure 5 and Figure 6) but this time with the shelf response's instantaneous frequencies revealed an increase in average by 2.19 CPD (Hilbert Method) and 3.75 CPD (Quad Method).

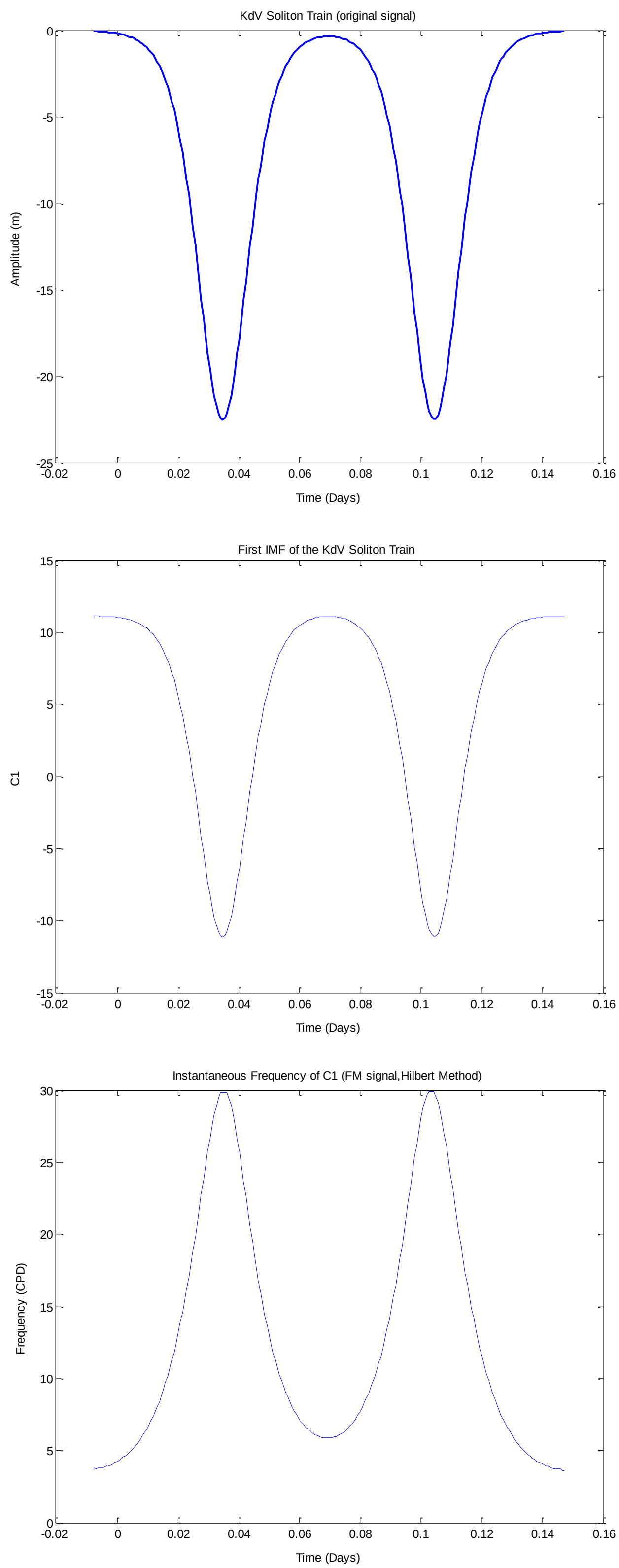


Figure 1. Original offshore KdV soliton train, peaks separated by 100 minutes (Top). First IMF (C1) obtained by EMD analysis (Middle) and C1's Instantaneous Frequency (Bottom).

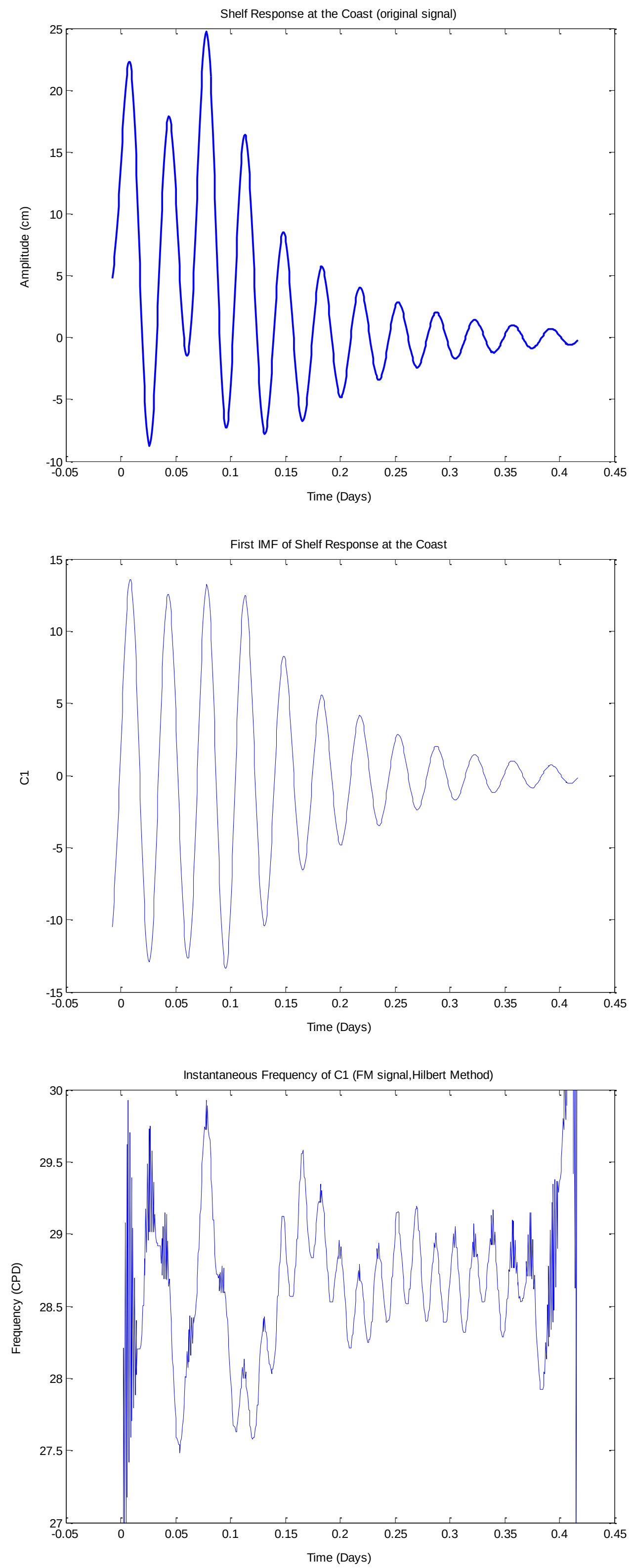
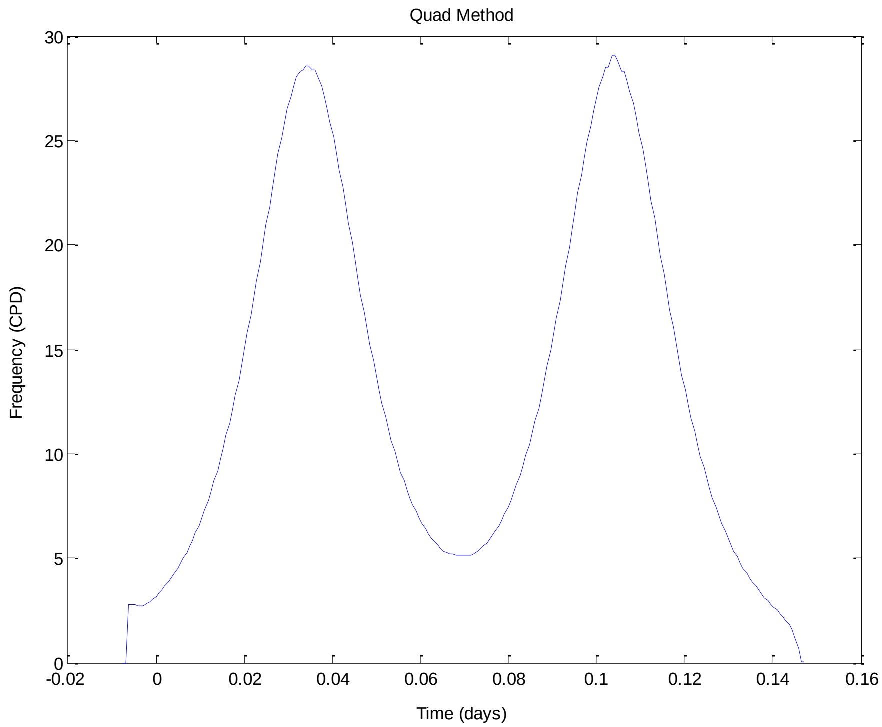


Figure 2. Coastal Shelf Response generated by a KdV soliton train (Top). C1, first IMF obtained by EMD analysis of the Shelf Response (Middle) and C1's Instantaneous Freq. by the Hilbert Method (Bottom).



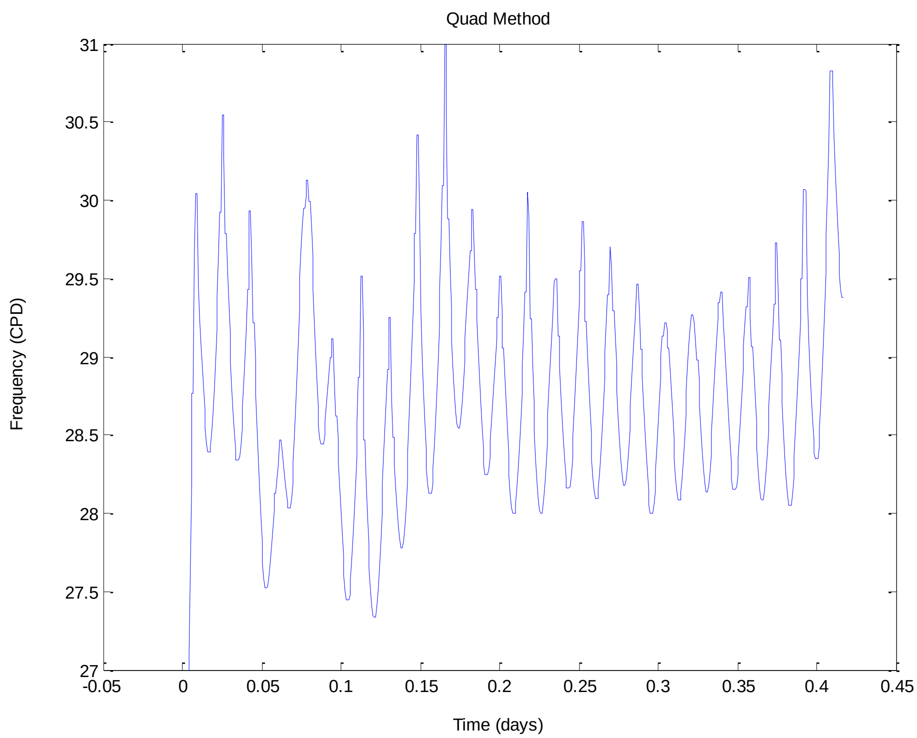
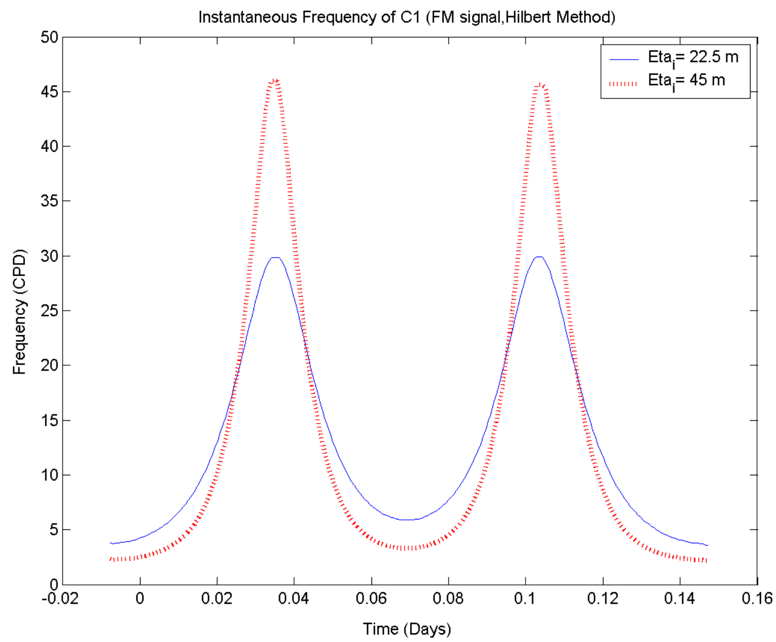
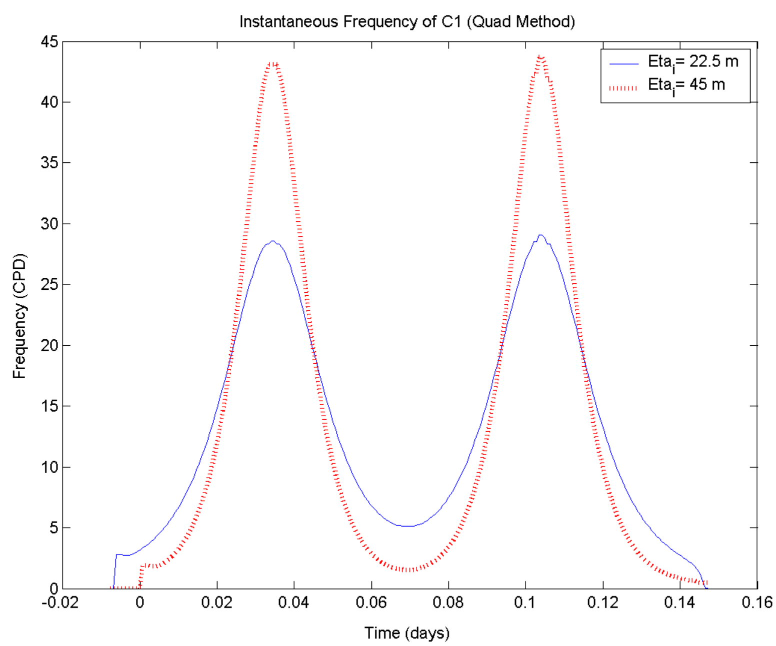


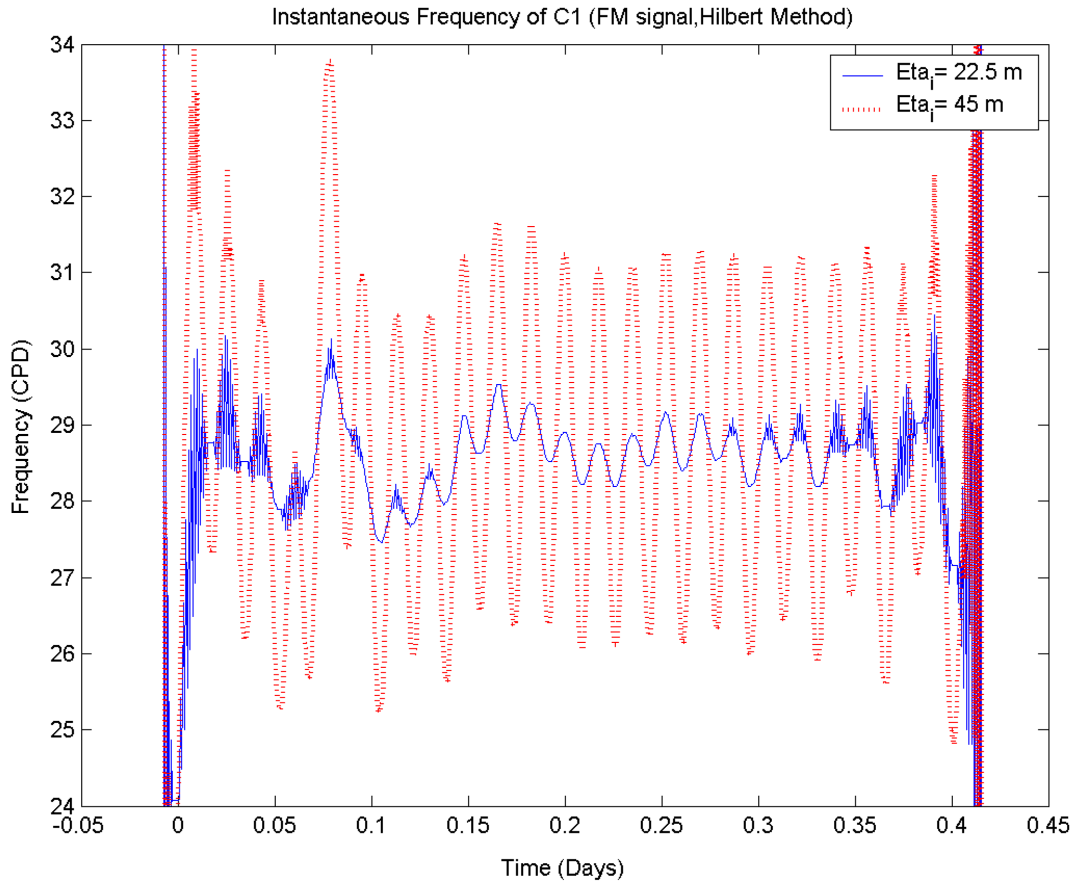
Figure 3. KdV Soliton Train's Instantaneous Frequency (Top). Shelf Response's Instantaneous Frequency (Bottom). Both Instantaneous Frequencies were obtained by the Quad Method.





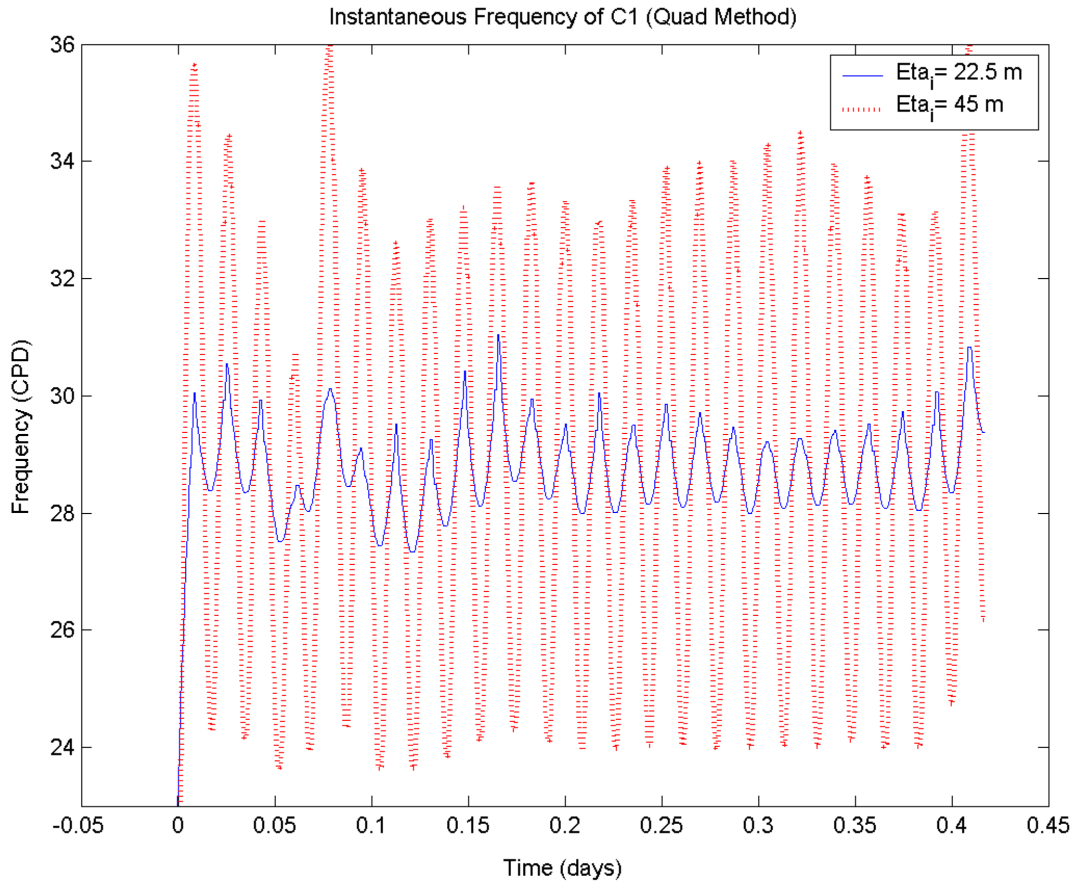
Time (Day)	C1's IF (CPD) ETA_i = 22.5 m	C1's IF (CPD) ETA_i = 45 m	Difference (CPD)	Method
0.035	29.90	45.98	16.08	Hilbert
0.035	28.49	43.09	14.60	Quad
	Mean Difference		**15.34**	

Figure 4. Comparison between C1's instantaneous frequencies once doubled the soliton amplitude from 22.5 m to 45 m. Hilbert Method (Top) and Quad Method (Bottom).



Time (Day)	C1's IF (CPD) ETA_i = 22.5 m	C1's IF (CPD) ETA_i = 45 m	Difference (CPD)
0.1133	28.05	30.42	2.37
0.1312	28.36	30.38	2.02
0.1487	29.11	31.17	2.06
0.1653	29.52	31.62	2.09
0.1818	29.28	31.60	2.32
0.1998	28.89	31.22	2.34
0.2177	28.74	31.02	2.28
0.2352	28.85	31.04	2.19
0.2527	29.15	31.21	2.06
0.2702	29.13	31.26	2.13
	Mean Difference		**2.19**

Figure 5. Comparison between the Shelf Response's instantaneous frequencies once doubled the soliton amplitude from 22.5 m to 45 m. Hilbert Method.



Time (Day)	C1's IF (CPD) ETA_i = 22.5 m	C1's IF (CPD) ETA_i = 45 m	Difference (CPD)
0.1128	29.50	32.59	3.09
0.1303	29.23	33.02	3.79
0.1478	30.40	33.17	2.77
0.1653	30.98	33.56	2.58
0.1832	29.91	33.63	3.72
0.2007	29.50	33.32	3.82
0.2177	30.03	32.95	2.92
0.2352	29.48	33.27	3.79
0.2532	29.82	33.83	4.01
0.2697	29.65	33.92	4.28
0.2877	29.43	34.02	4.59
0.3047	29.18	34.19	5.01
	Mean Difference		**3.75**

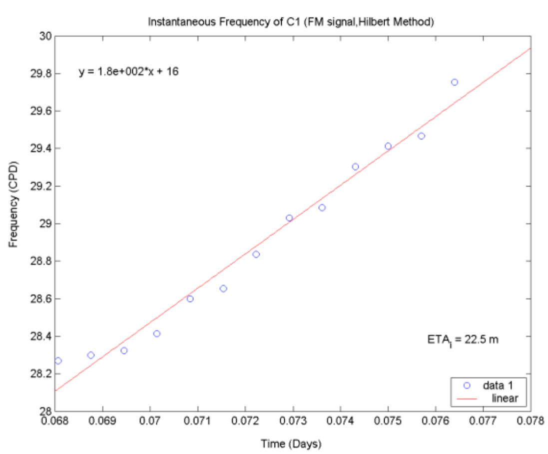
Figure 6. Comparison between Shelf Response's instantaneous frequencies once doubled the soliton amplitude from 22.5 m to 45 m. Quad Method.

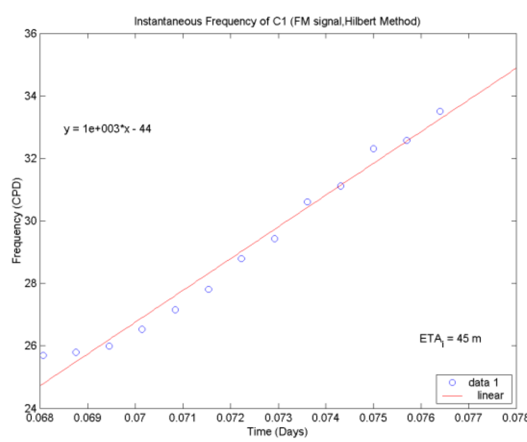
Discussion

Our results demonstrate that doubling the amplitude of a KdV soliton train increases dramatically the peak instantaneous frequency. The shelf response's instantaneous frequency also increases but in a lesser degree. Another interesting finding is that the KdV soliton arrival can manifest as a peak jump in the shelf response's instantaneous frequency. In addition, it widens the peak base. These findings provide us with key information that helps us interpret the signal in their physical terms. This is the goal of *Seichelogía* (Alfonso-Edwin, 2013); to determine the physical properties of the soliton and oceanic medium based only in the careful analysis of the nonlinear coastal seiche signal.

Comparing the results from the Hilbert and Quad methods, it seems that the Quad method is less noisy at the edges but seems to overshoot the maximum peaks and minimum valleys through the whole signal. To the contrary, the Hilbert method is noisier at the edges but is peak values are closer to the real frequency values. How do we know that? The maximum shelf response at La Parguera Shelf occurs at the non-dimensional frequency of 0.105 equal to 28.8 CPD. This is the natural frequency based on the shelf's length and depth. In average, the Hilbert method surpasses the shelf's natural frequency by 2 CPD but the Quad method overshoots that value for up to 4 CPD.

Another important finding is that doubling the KdV soliton amplitude increases the instantaneous frequency slope. The figure below shows the instantaneous frequency from 0.068 days to 0.078 days for each case. For the first case when the soliton amplitude is 22.5 m the slope is 180 CPD/D but when the amplitude is doubled to 45 m the new slope value is 1000 CPD/D. The instantaneous frequencies slopes obtained for large amplitude seiches in La Parguera range between 70 CPD/D and 490 CPD/D.





Conclusion

An increase in amplitude of a KdV soliton train increases the instantaneous frequency peaks and slopes for both nonlinear signals: the KdV soliton train and the coastal shelf response.

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